Super-Resolution
(and other promising mathematical methods for image analysis)

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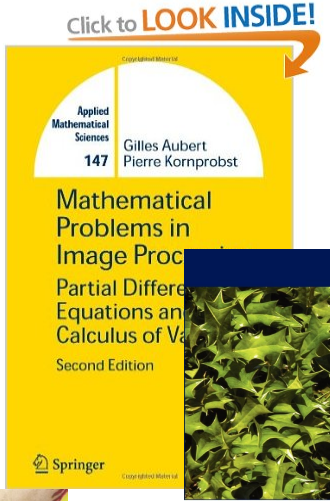
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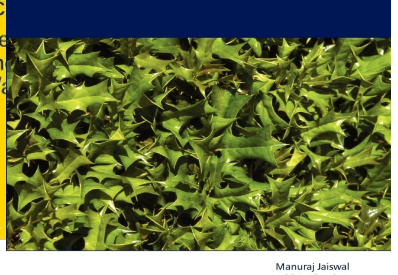
Good news: many other fields (bio-med, forensic, video industry) have problems similar to those of astronomers... From dealing with poor resolution to object segmentation.

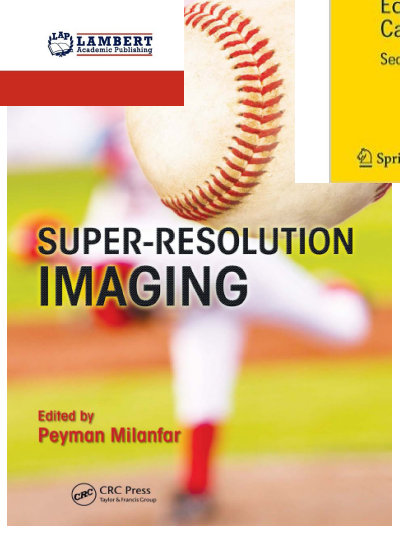
The field of image processing is a well developed branch of applied mathematics: in the last ~20 years have proposed image analysis techniques with roots (mainly) in the theory of:

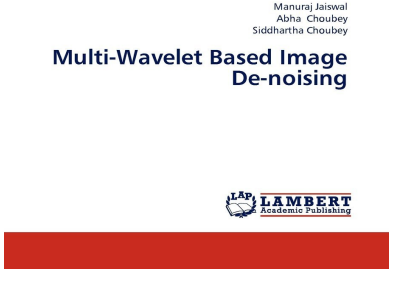
1) Calculus of Variations
2) Partial Differential Equations
3) Wavelet Transforms

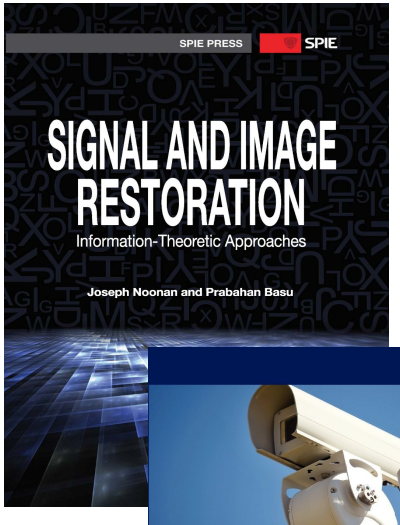




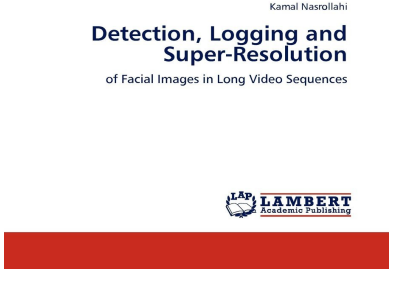


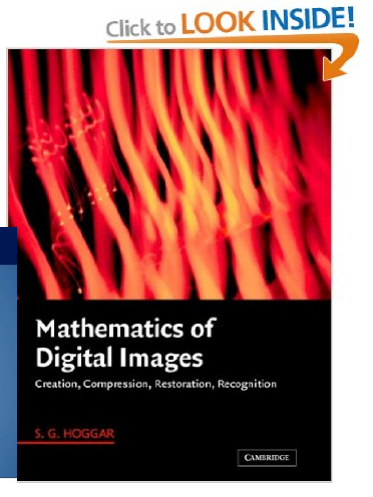






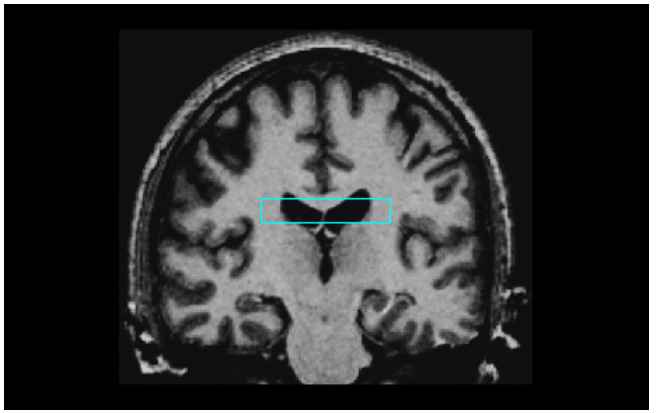






Some techniques that might be promising for astronomical images:

- *Active contour segmentation* (“level-set methods”, e.g. Sethian 1996)



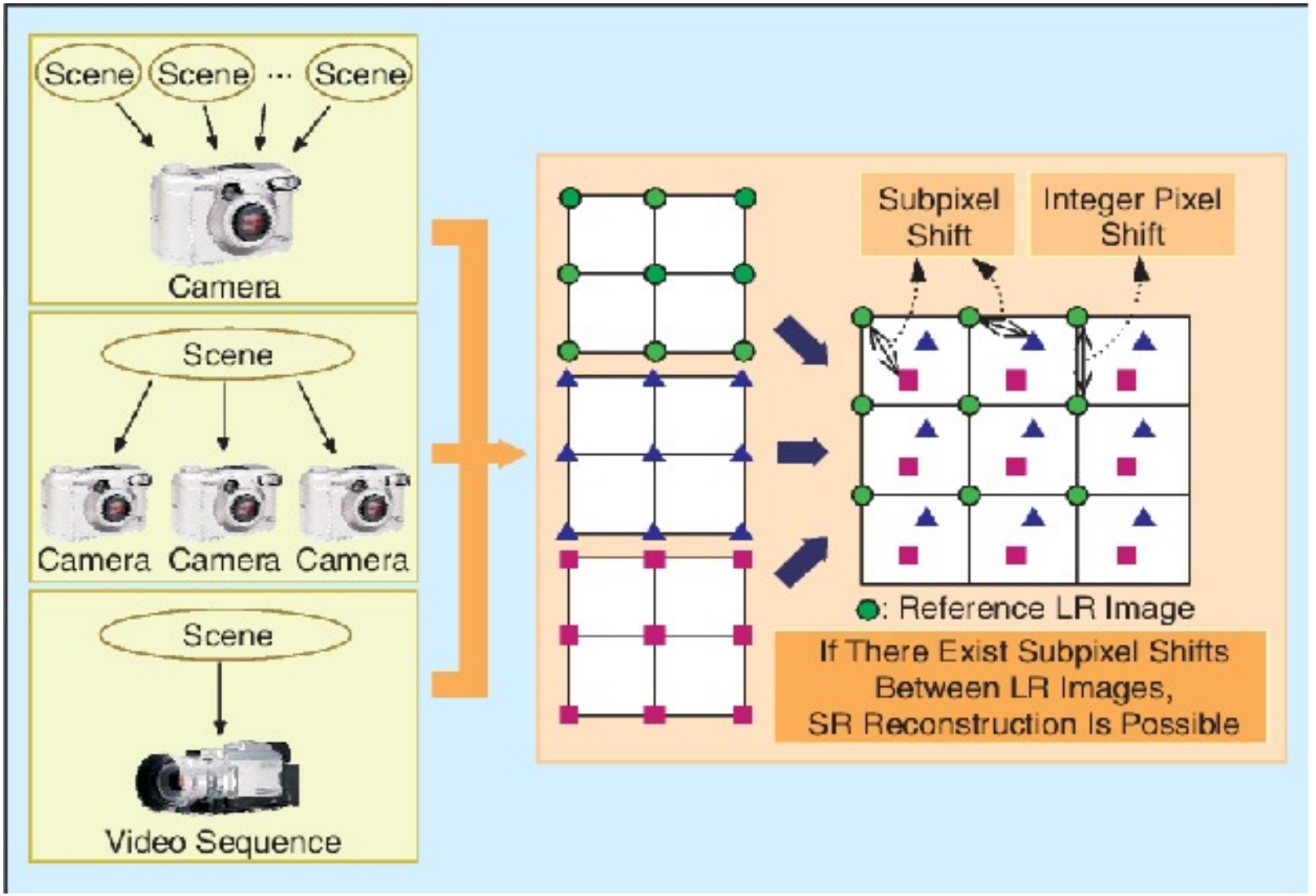
- *Structure-texture decomposition* (e.g. Rudin et al. 1992)



- ***Super-resolution*** (e.g. Irani&Peleg 1991)

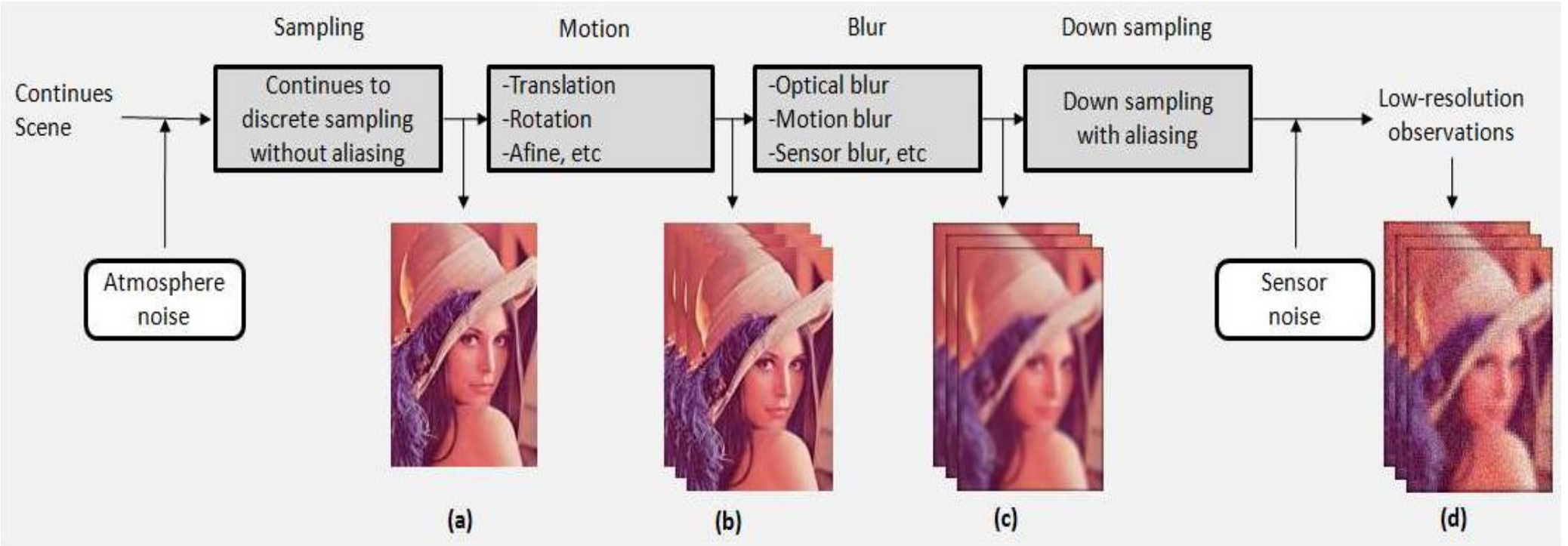


What is Super-Resolution ?



Given a set of “LR frames” with sub-pixel shifts between them we can reconstruct an higher resolution image. BUT: SR is not (only) an interpolation technique!

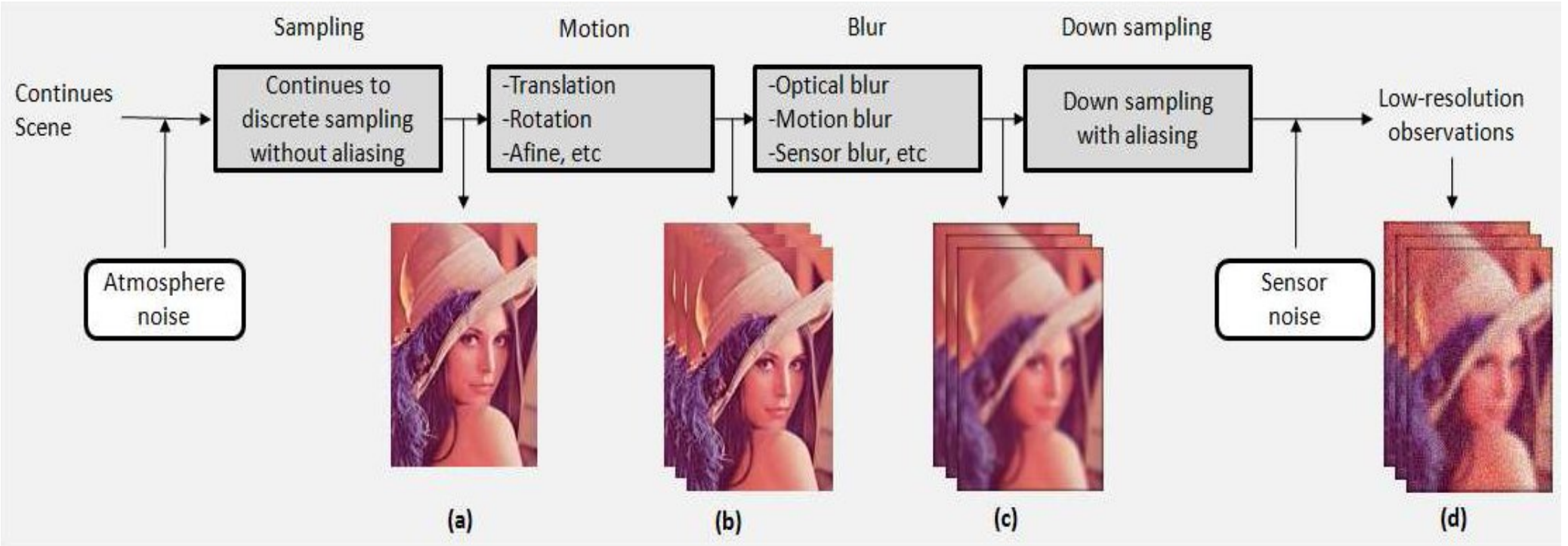
Super-Resolution:
Image formation model



$$\underline{x}_L^{(n)} = DH_n W_n \underline{x}_H + \underline{\epsilon}_n$$

Low resolution images can be considered as the application of warping (W), convolution (H) and downsampling (D) operators on the high-res frame.

Super-Resolution:
Image formation model



$$\begin{bmatrix} \underline{x}_L^{(1)} \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}_L^{(p)} \end{bmatrix} = \begin{bmatrix} DH_1W_1 \\ \cdot \\ \cdot \\ \cdot \\ DH_pW_p \end{bmatrix} \underline{x}_H + \begin{bmatrix} \underline{\epsilon}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{\epsilon}_p \end{bmatrix} \Longleftrightarrow \underline{x}_L = K\underline{x}_H + \epsilon$$

Super-Resolution techniques

Different approaches:

- Fourier-based techniques (Tsai&Huang 1984)

$$\underline{\tilde{y}} = \Phi \underline{\tilde{X}}_H$$

- Variational methods (e.g. Hardie et al. 1998, Mitzel et al. 2009)

$$E(\underline{x}_H) = \frac{1}{2}(\|\underline{x}_L - K\underline{x}_H\|_2^2 + \lambda f(\underline{x}_H))$$

- Bayesian approaches (e.g. Pickup et al. 2009)

$$X_H = \arg\max_{x_H} P(x_H|x_L) = \arg\max_{x_H} \frac{P(x_L|x_H)P(x_H)}{P(x_L)}$$

- “Example-based” and “image hallucination” approaches (e.g. Datsenko&Elad 2007, Glasner et al. 2009).

Variational Algorithms for image SR (1)

Regularization term given by the L2 norm of the image
(e.g. Mitzel et al. 2009, Zomet&Peleg 2000, Hardie et al. 1998)

$$E(\underline{x}_H) = \frac{1}{2}(||\underline{x}_L - K\underline{x}_H||_2^2 + \lambda||\nabla\underline{x}_H||_2^2)$$

It's a convex function, the minimum is given by the condition:

$$\nabla E(\underline{x}_H) = K^T(K\underline{x}_H - x_L) - \lambda\Delta\underline{x}_H = \sum_{n=1}^{N} W_n^T H_n^T D^T (DH_n W_n \underline{x}_H - x_L) - \lambda\Delta\underline{x}_H = 0$$

Steepest descent minimization:

$$\underline{x}_H^{m+1} = \underline{x}_H^m - \beta \cdot \nabla E(\underline{x}_H) = \underline{x}_H^m - \beta \cdot [\sum_{n=1}^{N} W_n^T H_n^T D^T (DH_n W_n \underline{x}_H^m - x_L) - \lambda\Delta\underline{x}_H^m]$$

With step: $\frac{1}{\beta} < L \leq |\lambda|||\Delta|| + ||K^T K|| = 8|\lambda| + 1$

Variational Algorithms for image SR (2)

Regularization term given by the L1 norm of the image (Unger et al. 2010). ("edge-preserving" properties)

$$E(\underline{x}_H) = ||\underline{x}_L - K\underline{x}_H||^r_{\epsilon_d} + \lambda||\nabla \underline{x}_H||_{\epsilon_u}$$

Based on the Huber (1980) norm:

$$||\underline{x}||^h_\epsilon = \sum_{0 \le i,j \le MN} |x_{i,j}|_\epsilon h^2$$

with:

$$|x_{i,j}|_\epsilon = \frac{|x_{i,j}|^2}{2\epsilon} \qquad \text{se } |x_{i,j}| \le \epsilon$$

$$|x_{i,j}|_\epsilon = |x_{i,j}| - \frac{\epsilon}{2} \qquad \text{se } |x_{i,j}| > \epsilon$$

$$\epsilon = \frac{1.345 \cdot MAD}{0.6746}$$

MAD="mean absolute deviation"=median(x-median(x))

Test on pictures



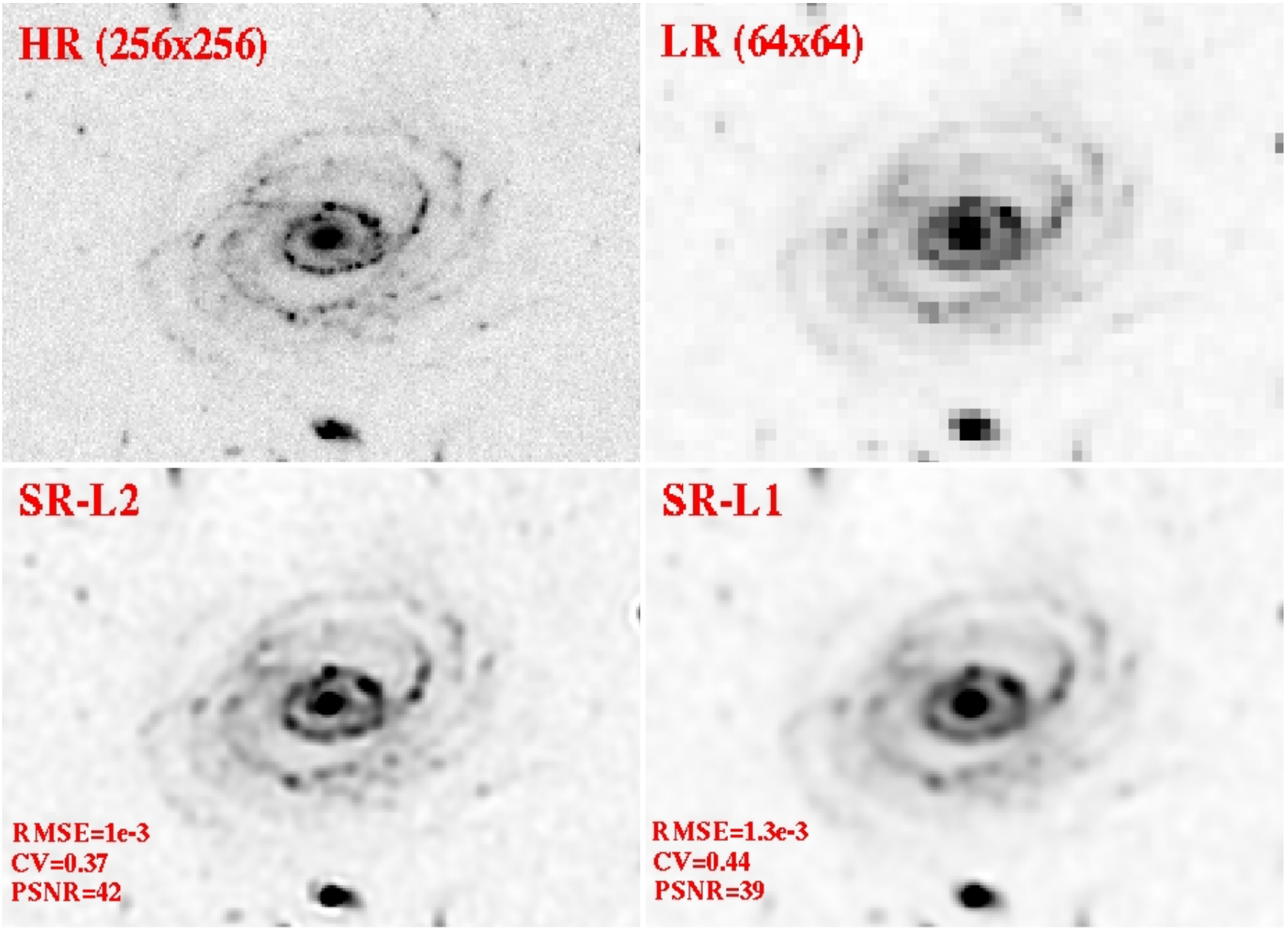
(SR-L2)

(SR-L1)

Test on pictures

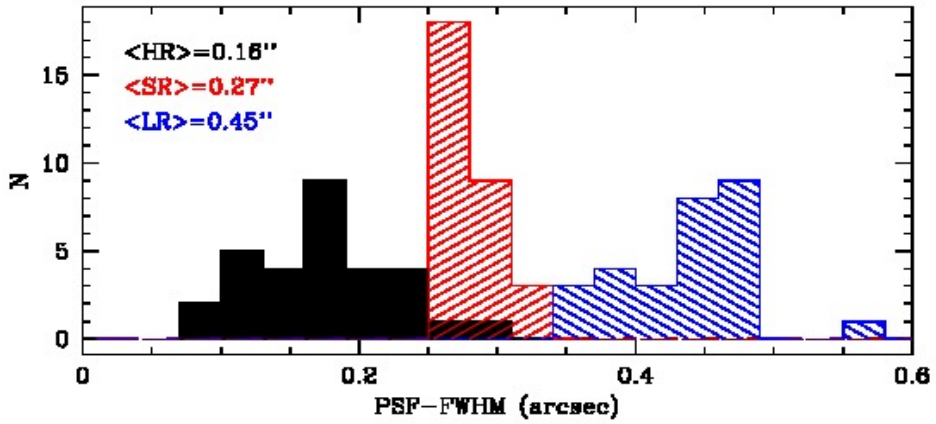


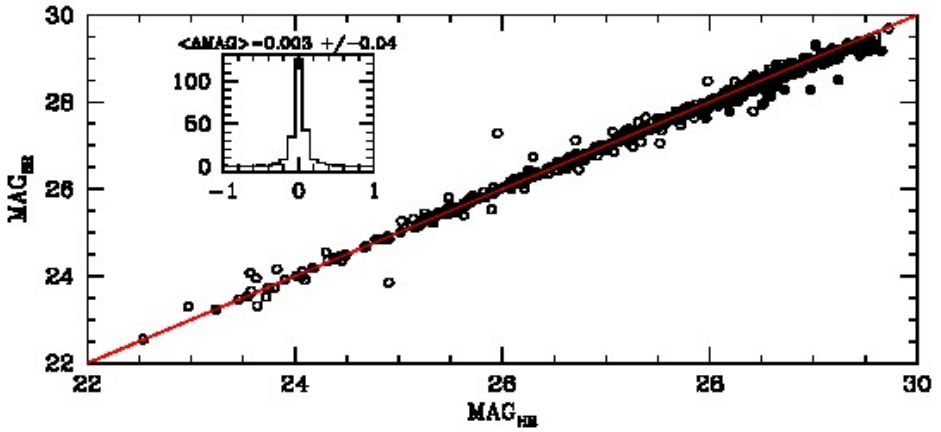
Simulations using HUDF



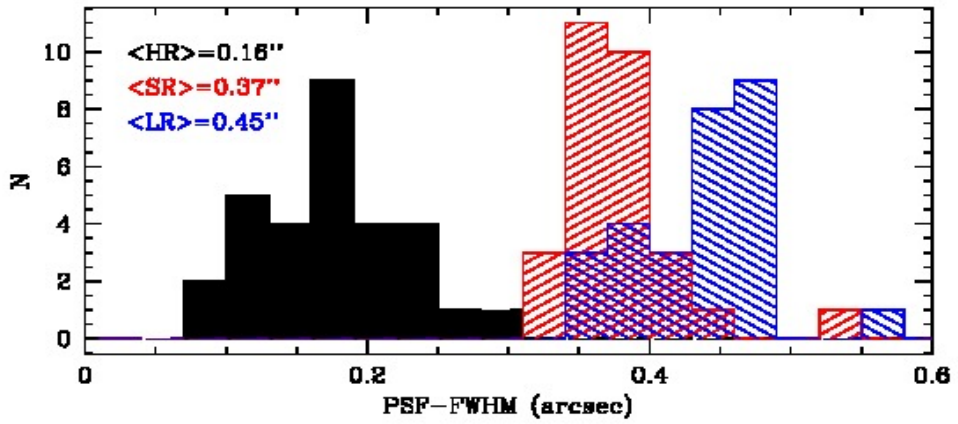
Simulations using HUDF: photometry

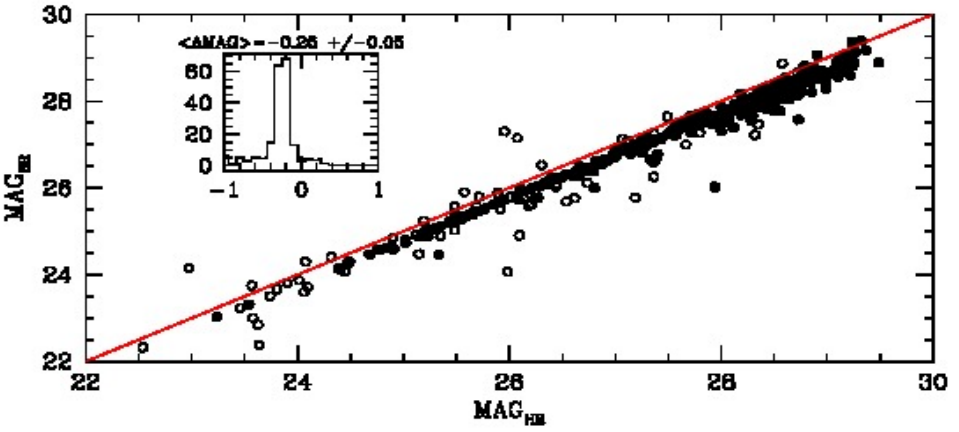
SR-L2



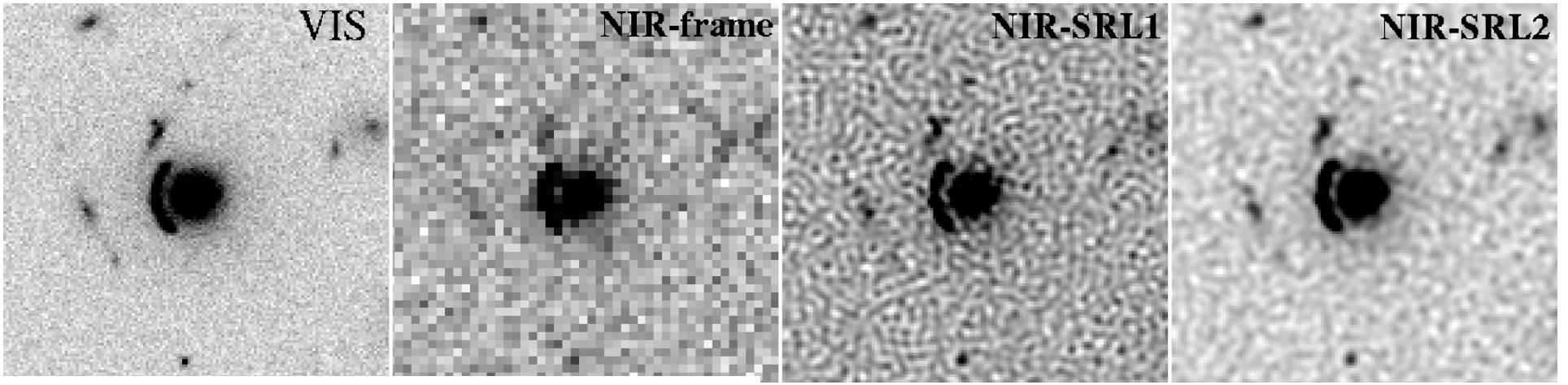


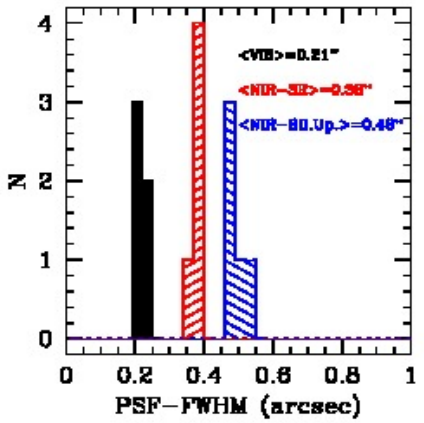
SR-L1

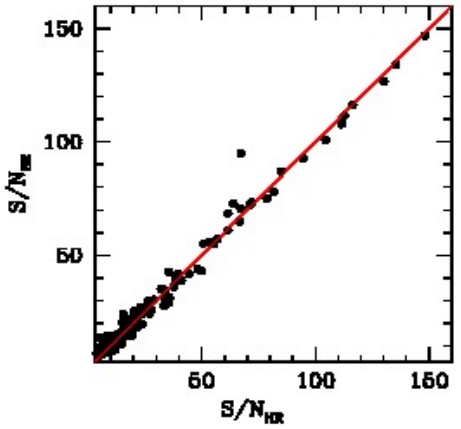


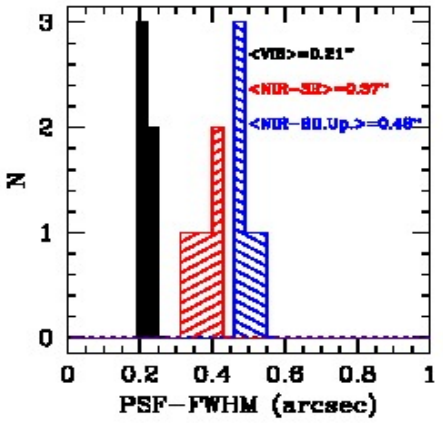


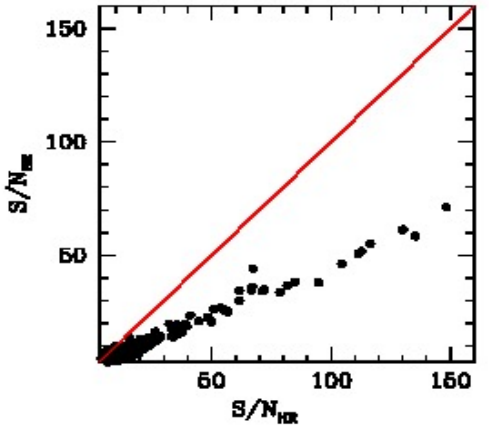
EUCLID Simulations

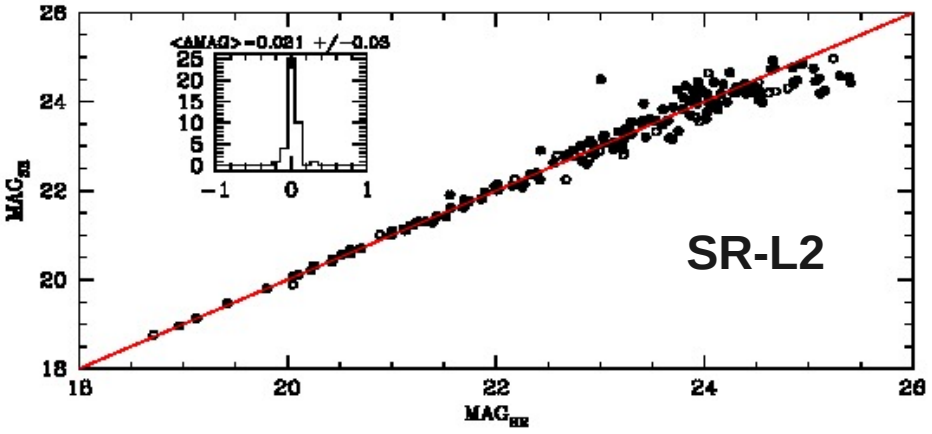


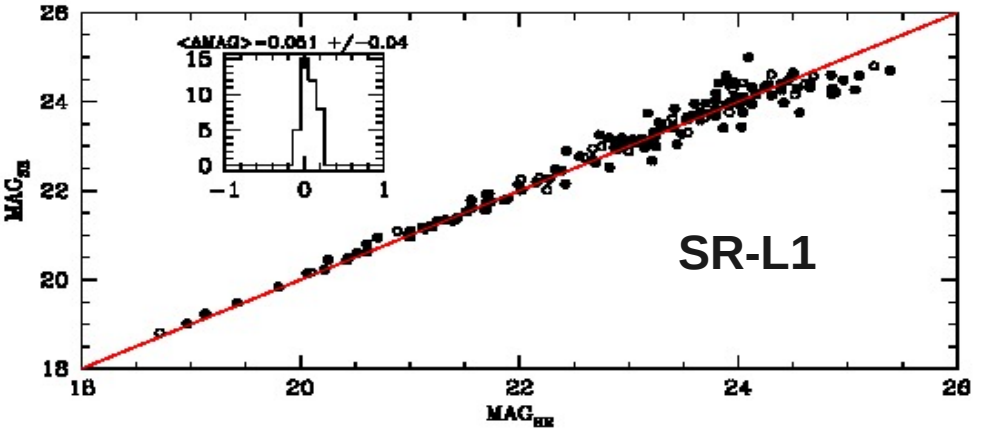








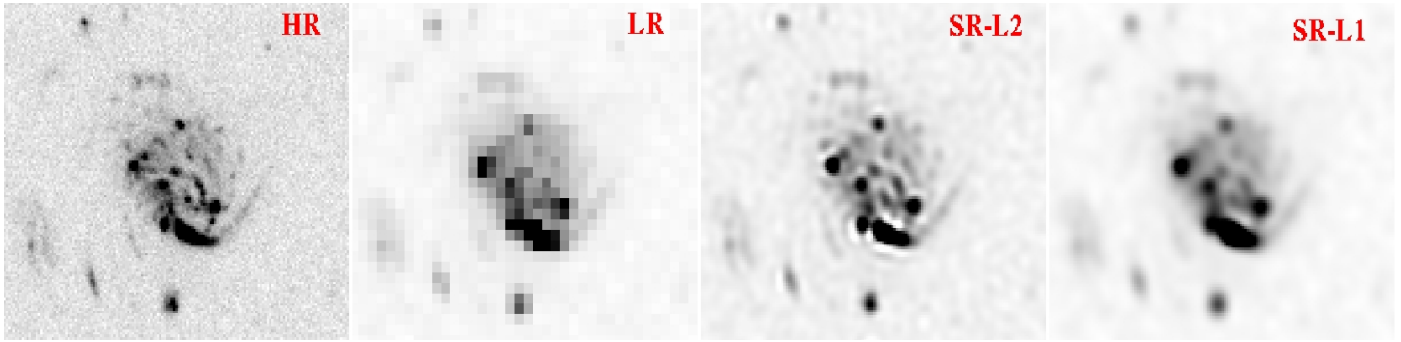




CONCLUSIONS (1)

- SR is not an interpolation technique: totally different approach that aims at both “drizzling” and deconvolve a set of images.

- Simple tests on HUDF and on our EUCLID simulations are promising: deconvolution is effective, calibration is not altered, noise reduced as through a coaddition (SR-L2).



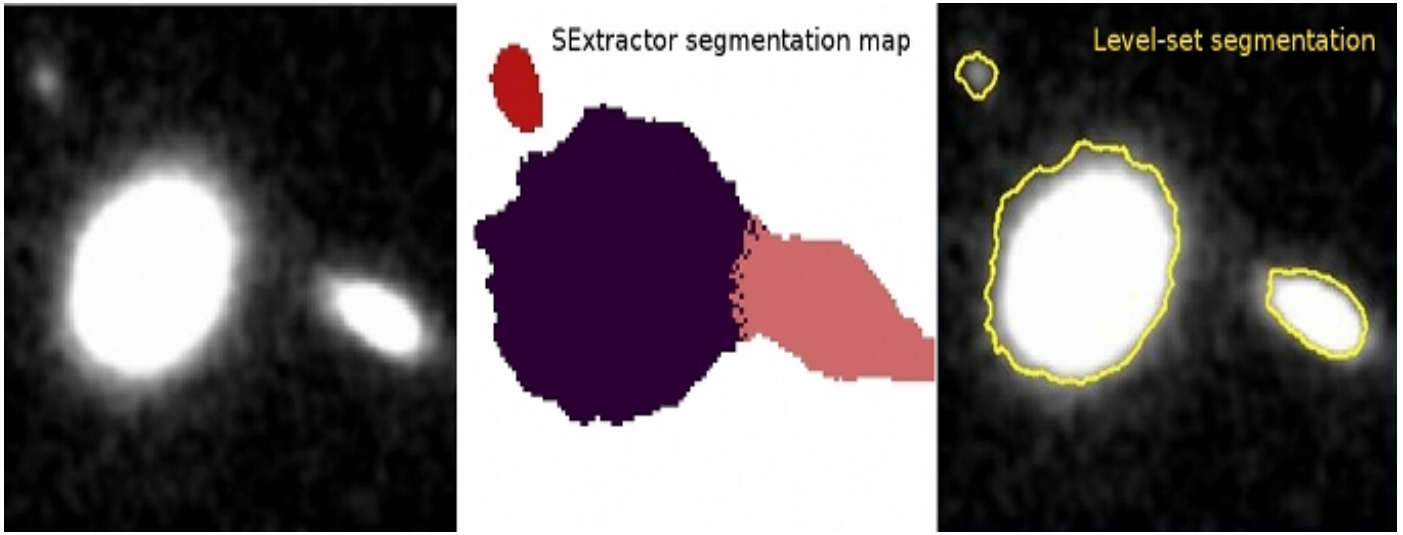
- The goal is to fully develop an “Astro-SR” as a valuable aid for multi-lambda detection, deblending etc.

Other tests needed: work in progress.

CONCLUSIONS (2)

The feasibility of the other techniques in the astronomical context must be assessed!

- Level-set segmentation



(output of a MATLAB script working on a JPEG thumbnail from GOODS-S)

- Structure-texture decomposition might be useful for estimating sky background and/or object detection...

